

Student Workbook

Inverses & Radical Functions

Learning Target: I can find inverses of restricted quadratic and radical functions, and solve radical equations while checking for extraneous solutions.

Vocabulary

Term	Meaning
Radical Function	A function involving a root, such as $f(x) = \sqrt{x}$.
Extraneous Solution	A value obtained algebraically that does NOT satisfy the original equation — common after squaring both sides.

Key Concept: Inverse of a Restricted Quadratic

WHY THE RESTRICTION? $y=x^2$ is not one-to-one over all reals, so it has no inverse function unless we restrict the domain (usually $x \geq 0$).

EXAMPLE 1

Find the inverse of $f(x) = x^2 + 3$, $x \geq 0$.

Swap: $x = y^2 + 3 \rightarrow y^2 = x - 3 \rightarrow y = \underline{\hspace{2cm}}$ (positive root, since range must match original domain $x \geq 0$)

Domain of f^{-1} : $\underline{\hspace{2cm}}$

Key Concept: Inverse of a Radical Function

EXAMPLE 2

Find the inverse of $f(x) = \sqrt{x-2} + 1$.

Swap: $x = \sqrt{y-2} + 1 \rightarrow x-1 = \sqrt{y-2} \rightarrow (x-1)^2 = y-2$

$f^{-1}(x) = \underline{\hspace{2cm}}$, domain $x \geq \underline{\hspace{2cm}}$

Key Concept: Solving Radical Equations

METHOD 1. Isolate the radical. 2. Raise both sides to the matching power (square for a square root). 3. Solve. 4. ALWAYS check each solution in the ORIGINAL equation for extraneous solutions.

EXAMPLE 3

Solve $\sqrt{x+4} = x-2$.

Square: $x+4 = x^2-4x+4 \rightarrow 0 = x^2-5x \rightarrow x(x-5)=0 \rightarrow x=0$ or $x=5$

Check $x=0$: $\sqrt{4} = 2$, but $x-2 = -2$. _____ (extraneous)

Check $x=5$: $\sqrt{9} = 3$, and $x-2 = 3$. _____ (valid)

COMMON MISTAKE

Skipping the extraneous solution check! Squaring both sides can introduce solutions that don't actually work in the original equation — ALWAYS check.

Guided Practice — Part A: Inverse of a Restricted Quadratic

A1 $f(x) = x^2 - 4, x \geq 0$

A2 $f(x) = (x-1)^2 + 5, x \geq 1$

Guided Practice — Part B: Inverse of a Radical Function

B1 $f(x) = \sqrt{x+6} - 3$

B2 $f(x) = \sqrt{2x-4}$

Guided Practice — Part C: Solve Radical Equations

C1 $\sqrt{3x+1} = 4$

C2 $\sqrt{x+10} = x-2$

Guided Practice — Part D: Domain & Range

D1 $f(x) = \sqrt{x-5} + 2$

D2 $f(x) = -\sqrt{x+3} + 4$

BEFORE YOU MOVE ON

Check with a partner: for every radical equation, did you plug your solution(s) back into the ORIGINAL equation to check?